

Totally β^* - Continuous Functions in Topological Spaces

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Abstract

The aim of this paper is to define a new class of functions namely totally β^* - continuous functions and slightly β^* - continuous functions and study their properties . Additionally, we relate and compare these functions with some other functions in topological spaces.

Keywords and phrases: Totally β^* - continuous and Slightly β^* - continuous.

I. Introduction

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. Abd El- Monsef et al. introduced the notion of β - open sets and β -continuity in topological spaces. RC Jain introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper, we define totally β^* - continuous functions and slightly β^* - continuous functions and basic properties of these functions are investigated and obtained.

II. Preliminaries

Throughout this paper (X, τ) , (Y, σ) and (Z, η) or X, Y, Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure and the interior of A respectively. The power set of X is denoted by $P(X)$. If A is β^* -open and β^* - closed , then it is said to be β^* - clopen.

Definition 2.1: A subset A of a topological space X is said to be a β^* -open [5] if $A \subseteq \text{cl} (\text{int}^* (\text{cl}(A)))$.

Definition 2.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called totally continuous [2] if $f^{-1} (V)$ is clopen set in X for each open set V of Y .

Definition 2.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a β^* - continuous [8] if $f^{-1} (O)$ is a β^* -open set of (X, τ) for every open set O of (Y, σ) .

Definition 2.4: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly β^* -continuous [6] if the inverse image of every β^* -open set in (Y, σ) is both open and closed in (X, τ) .

Definition 2.5: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a slightly continuous [2] if the inverse image of every clopen set in Y is open in X .

Definition 2.6: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a contra continuous [1] if $f^{-1}(O)$ is closed in (X, τ) for every open set O in (Y, σ) .

Definition 2.7: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called Contra β^* -continuous functions [7] if $f^{-1}(O)$ is β^* -closed in (X, τ) for every open set O in (Y, σ) .

Definition 2.8: A topological space X is called a β^* -connected [9] if X cannot be expressed as a disjoint union of two non-empty β^* -open sets.

Definition 2.9: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be pre β^* -open [7] if the image of every β^* -open set of X is β^* -open in Y .

Definition 2.10: A topological space X is said to be connected [10] if X cannot be expressed as the union of two disjoint nonempty open sets in X .

Definition 2.11: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a strongly β^* -continuous [6] if the inverse image of every β^* -open set in (Y, σ) is open in (X, τ) .

Definition 2.12: A Topological space X is said to be β^* - $T_{1/2}$ space or β^* -space [8] if every β^* -open set of X is open in X .

Definition 2.13: A space (X, τ) is called a locally indiscrete space [3] if every open set of X is closed in X .

Theorem 2.14[5]:

(i) Every open set is β^* -open and every closed set is β^* -closed set.

III. Totally β^* -continuous functions

Definition 3.1: A function $(X, \tau) \rightarrow (Y, \sigma)$ is called totally β^* -continuous functions if the inverse image of every open set of (Y, σ) is both β^* -open and β^* -closed subset of (X, τ) .

Example 3.2: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$, $\beta^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\beta^*C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. since, $f^{-1}(\{a\}) = \{b\}$, $f^{-1}(\{a, b\}) = \{b, c\}$ and $f^{-1}(\{a, c\}) = \{a, b\}$ is both β^* -open and β^* -closed in X . Therefore, f is totally β^* -continuous.

Theorem 3.2: Every totally β^* -continuous functions is β^* -continuous.

Proof: Let O be an open set of (Y, σ) . Since, f is totally β^* -continuous functions, $f^{-1}(O)$ is both β^* -open and β^* -closed in (X, τ) . Therefore, f is β^* -continuous.

Remark 3.3: The converse of above theorem need not be true.

Example 3.4: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. $\beta^*O(X, \tau) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\beta^*C(X, \tau) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly, f is not totally β^* -continuous since $f^{-1}(\{a, b\}) = \{a, b\}$ is β^* -open in X but not β^* -closed. However, f is β^* -continuous.

Theorem 3.5: Every totally continuous function is totally β^* -continuous.

Proof: Let O be an open set of (Y, σ) . Since, f is totally continuous functions, $f^{-1}(O)$ is both open and closed in (X, τ) . Since every open set is β^* -open and every closed set is β^* -closed. $f^{-1}(O)$ is both β^* -open and β^* -closed in (X, τ) . Therefore, f is totally β^* -continuous.

Remark 3.6: The converse of above theorem need not be true.

Example 3.7: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$, $\tau^c = \{\emptyset, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. $\beta^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$, $\beta^*C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Clearly, f is totally β^* -continuous but $f^{-1}(\{a, b\}) = \{a, b\}$, $f^{-1}(\{a, c\}) = \{a, c\}$ is not open and closed in X . Therefore, f is not totally continuous.

Theorem 3.8: Every perfectly β^* -continuous map is totally β^* -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a perfectly β^* -continuous map. Let O be an open set of (Y, σ) . Then O is β^* -open in (Y, σ) . Since f is perfectly β^* -continuous, $f^{-1}(O)$ is both open and closed in (X, τ) , implies $f^{-1}(O)$ is both β^* -open and β^* -closed in (X, τ) . Therefore, f is totally β^* -continuous.

Remark 3.9: The converse of above theorem need not be true.

Example 3.10: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, X\}$, $\tau^c = \{\emptyset, \{c, d\}, \{d\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b, c, d\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. $\beta^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$, $\beta^*C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. $\beta^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$. Clearly, f is totally β^* -continuous but $f^{-1}(\{a\}) = \{a\}$, $f^{-1}(\{b\}) = \{b\}$, $f^{-1}(\{c\}) = \{c\}$, $f^{-1}(\{d\}) = \{d\}$, $f^{-1}(\{a, c\}) = \{a, c\}$, $f^{-1}(\{a, d\}) = \{a, d\}$, $f^{-1}(\{b, c\}) = \{b, c\}$, $f^{-1}(\{b, d\}) = \{b, d\}$, $f^{-1}(\{c, d\}) = \{c, d\}$, $f^{-1}(\{a, b, d\}) = \{a, b, d\}$, $f^{-1}(\{a, c, d\}) = \{a, d\}$, $f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not open and closed in X . Therefore, f is not perfectly β^* -continuous.

Remark 3.11: The concept of totally β^* -continuous and strongly β^* -continuous are independent of each other.

Example 3.12: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, b\}, X\}$, $\tau^c = \{\emptyset, \{c, d\}, X\}$, $\beta^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$, $\beta^*C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{abc\}, Y\}$. $\beta^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a,$

$b, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$ Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Clearly, f is totally β^* -continuous but $f^{-1}(\{b\}) = \{b\}, f^{-1}(\{c\}) = \{c\}, f^{-1}(\{d\}) = \{d\}, f^{-1}(\{a, c\}) = \{a, c\}, f^{-1}(\{a, d\}) = \{a, d\}, f^{-1}(\{b, c\}) = \{b, c\}, f^{-1}(\{b, d\}) = \{b, d\}, f^{-1}(\{c, d\}) = \{c, d\}, f^{-1}(\{a, b, c\}) = \{a, b, c\}, f^{-1}(\{a, b, d\}) = \{a, b, d\}, f^{-1}(\{a, c, d\}) = \{a, c, d\}, f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not open in X . Therefore, f is not strongly β^* -continuous.

Example 3.13: Let $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}, \tau^c = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}, \sigma = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = b, f(c) = a$. $\beta^* O(X, \tau) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}, \beta^* C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\beta^* O(Y, \sigma) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Clearly, f is strongly β^* -continuous but $f^{-1}(\{a\}) = \{c\}, f^{-1}(\{a, b\}) = \{b, c\}, f^{-1}(\{a, c\}) = \{a, c\}$ is β^* -open in X but not β^* -closed. Therefore, f is not totally β^* -continuous.

Theorem 3.14: If $f: X \times Y$ is a totally β^* -continuous map, and X is β^* -connected, then Y is an indiscrete space.

Proof: Suppose that Y is not an indiscrete space. Let A be a non-empty open subset of Y . Since, f is totally β^* -continuous map, then $f^{-1}(A)$ is a non-empty β^* -clopen subset of X . Then $X = f^{-1}(A) \cup (f^{-1}(A))^c$. Thus, X is a union of two non-empty disjoint β^* -open sets which is contradiction to the fact that X is β^* -connected. Therefore, Y must be an indiscrete space

Theorem 3.15: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then $g \circ f: X \rightarrow Z$

- (i) If f is β^* -irresolute and g is totally β^* -continuous then $g \circ f$ is totally β^* -continuous
- (ii) If f is totally β^* -continuous and g is continuous then $g \circ f$ is totally β^* -continuous.

Proof:

(i) Let O be an open set in Z . Since g is totally β^* -continuous, $g^{-1}(O)$ is β^* -clopen in Y . Since f is β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open and β^* -closed in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Therefore, $g \circ f$ is totally β^* -continuous.

(ii) Let O be an open set in Z . Since g is continuous, $g^{-1}(O)$ is open in Y . Since, f is totally β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -clopen in X . Hence, $g \circ f$ is totally β^* -continuous.

IV. Slightly β^* -continuous functions.

Definition 4.1: A function $(X, \tau) \rightarrow (Y, \sigma)$ is called slightly β^* -continuous at a point $x \in X$ if for each clopen subset V of Y containing $f(x)$, there exists a β^* -open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly β^* -continuous if f is slightly β^* -continuous at each of its points.

Definition 4.2: A function $(X, \tau) \rightarrow (Y, \sigma)$ is said to be slightly β^* -continuous if the inverse image of every clopen set in Y is β^* -open in X .

Example 4.3: Let $X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, X\}, \sigma = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}, \sigma^c = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$, and $\beta^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}, \beta^* O(Y, \sigma) =$

$\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly, f is slightly β^* -continuous.

Proposition 4.4: The definition 4.1 and 4.2 are equivalent.

Proof: Suppose the definition 4.1 holds. Let O be a clopen set in Y and $x \in f^{-1}(O)$. Then $f(x) \in O$ and thus there exists a β^* -open set U_x such that $x \in U_x \subseteq f^{-1}(O)$ and $f^{-1}(O) = \bigcup U_x$. Since, arbitrary union of β^* -open set is β^* -open. $f^{-1}(O)$ is β^* -open in X and therefore, f is slightly β^* -continuous. Suppose, the definition 4.2 holds. Let $f(x) \in O$ where, O is a clopen set in Y . Since, f is slightly β^* -continuous, $x \in f^{-1}(O)$ where $f^{-1}(O)$ is β^* -open in X . Let $U = f^{-1}(O)$. Then U is β^* -open in X , $x \in X$ and $f(U) \subseteq O$.

Theorem 4.5: For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

- (i) f is slightly β^* -continuous.
- (ii) The inverse image of every clopen set O of Y is β^* -open in X .
- (iii) The inverse image of every clopen set O of Y is β^* -closed in X .
- (iv) The inverse image of every clopen set O of Y is β^* -clopen in X .

Proof:

(i) $\Rightarrow$ (ii): Follows from the proposition 4.4

(ii) $\Rightarrow$ (iii): Let O be a clopen set in Y which implies O^c is clopen in Y . By (ii), $f^{-1}(O^c) = (f^{-1}(O))^c$ is β^* -open in X . Therefore, $f^{-1}(O)$ is β^* -closed in X .

(iii) $\Rightarrow$ (iv): By (ii) and (iii), $f^{-1}(O)$ is β^* -clopen in X .

(iv) $\Rightarrow$ (i): Let O be a clopen set in Y containing $f(x)$, by (iv) $f^{-1}(O)$ is β^* -clopen in X . Take $U = f^{-1}(O)$, then $f(U) \subseteq O$. Hence, f is slightly β^* -continuous.

Theorem 4.6: Every slightly continuous function is slightly β^* -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a slightly continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is open in X . Since, every open set is β^* -open. Hence, f is slightly β^* -continuous.

Remark 4.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.8: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$, $\tau^c = \{\phi, \{d\}, \{b, c, d\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c, d\}, Y\}$, $\sigma^c = \{\phi, \{a\}, \{b, c, d\}, Y\}$ and $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. Clearly, f is slightly β^* -continuous but not slightly continuous. Since, $f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not open in X .

Theorem 4.9: Every β^* -continuous function is slightly β^* -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a β^* -continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is β^* -open in X and β^* -closed in X . Hence, f is slightly β^* -continuous.

Remark 4.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.11: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$, $\sigma^c = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$ and $\beta^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. The function f is slightly β^* -continuous but not β^* -continuous, since, $f^{-1} \{b\} = \{c\}$ is not β^* -open in X .

Theorem 4.12: Every contra β^* -continuous function is slightly β^* -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra β^* -continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is β^* -open in X . Hence, f is slightly β^* -continuous.

Remark 4.13: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.14: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$ and $\sigma^c = \{\emptyset, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$ and $\beta^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$, $\beta^* C(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. The function f is slightly β^* -continuous but not contra β^* -continuous, since, $f^{-1} \{(a, b, c)\} = \{a, b, c\}$ is not β^* -closed in X .

Remark 4.15: Composition of two slightly β^* -continuous need not be slightly β^* -continuous as it can be seen from the following example.

Example 4.16: Let $X = Y = Z = \{a, b, c, d\}$, and the topologies are $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$, $\sigma^c = \{\emptyset, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$ and $\eta = \{\emptyset, \{a\}, \{b, c, d\}, Z\}$, $\eta^c = \{\emptyset, \{a\}, \{b, c, d\}$. $\beta^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$, $\beta^* O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, Z\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. Clearly, f is slightly β^* -continuous. Define $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = a$, $g(b) = b$, $g(c) = c$, $g(d) = d$. Clearly, g is slightly β^* -continuous. But $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is not slightly β^* -continuous, since $(g \circ f)^{-1}(\{b, c, d\}) = f^{-1}(g^{-1}(\{b, c, d\})) = f^{-1}(\{b, c, d\}) = \{b, c, d\}$ is not a β^* -open in (X, τ) .

Theorem 4.17: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then the following properties hold:

- (i) If f is β^* -irresolute and g is slightly β^* -continuous then $(g \circ f)$ is slightly β^* -continuous.
- (ii) If f is β^* -irresolute and g is β^* -continuous then $(g \circ f)$ is slightly β^* -continuous.
- (iii) If f is β^* -irresolute and g is slightly continuous then $(g \circ f)$ is slightly β^* -continuous.

- (iv) If f is β^* -continuous and g is slightly continuous then $(g \circ f)$ is slightly β^* -continuous.
- (v) If f is strongly β^* -continuous and g is slightly β^* -continuous then $(g \circ f)$ is slightly continuous.
- (vi) If f is slightly β^* -continuous and g is perfectly β^* -continuous then $(g \circ f)$ is β^* -irresolute.
- (vii) If f is slightly β^* -continuous and g is contra continuous then $(g \circ f)$ is slightly β^* -continuous.
- (viii) If f is β^* -irresolute and g is contra β^* -continuous then $(g \circ f)$ is slightly β^* -continuous.

Proof:

- (i) Let O be a clopen set in Z . Since, g is slightly β^* -continuous, $g^{-1}(O)$ is β^* -open in Y . Since, f is β^* - β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$, $g \circ f$ is slightly β^* -continuous.
- (ii) Let O be a clopen set in Z . Since, g is β^* -continuous, $g^{-1}(O)$ is β^* -open in Y . Since, f is β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $g \circ f$ is slightly β^* -continuous.
- (iii) Let O be a clopen set in Z . Since, g is slightly continuous, $g^{-1}(O)$ is open in Y . Since, f is β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $g \circ f$ is slightly β^* -continuous.
- (iv) Let O be a clopen set in Z . Since, g is slightly continuous, $g^{-1}(O)$ is open in Y . Since, f is β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $g \circ f$ is slightly β^* -continuous.
- (v) Let O be a clopen set in Z . Since, g is slightly β^* -continuous, $g^{-1}(O)$ is β^* -open in Y . Since, f is strongly β^* -continuous, $f^{-1}(g^{-1}(O))$ is open in X . Therefore, $g \circ f$ is slightly continuous.
- (vi) Let O be a β^* -open in Z . Since, g is perfectly β^* -continuous, $g^{-1}(O)$ is open and closed in Y . Since, f is slightly β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $g \circ f$ is β^* -irresolute.
- (vii) Let O be a clopen set in Z . Since, g is contra continuous, $g^{-1}(O)$ is open and closed in Y . Since, f is slightly β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $g \circ f$ is slightly β^* -continuous.
- (viii) Let O be a clopen set in Z . Since, g is contra β^* -continuous, $g^{-1}(O)$ is β^* -open and β^* -closed in Y . Since, f is β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open and β^* -closed in X . Hence, $g \circ f$ is slightly β^* -continuous.

Theorem 4.18: If the function $f: (X, \tau) \rightarrow (Y, \sigma)$ is slightly β^* -continuous and (X, τ) is β^* - $T_{1/2}$ space, then f is slightly continuous.

Proof: Let O be a clopen set in Y . Since, g is slightly β^* -continuous, $f^{-1}(O)$ is β^* -open in X . Since, X is β^* - $T_{1/2}$ space, $f^{-1}(O)$ is open in X . Hence, f is slightly continuous.

Theorem 4.19: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be functions. If f is surjective and pre β^* -open and $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly β^* -continuous, then g is slightly β^* -continuous.

Proof: Let O be a clopen set in (Z, η) . Since, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Since, f is surjective and pre β^* -open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is β^* -open in Y . Hence, g is slightly β^* -continuous.

Theorem 4.20: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be functions. If f is surjective, pre β^* -open and β^* -irresolute, then $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly β^* -continuous if and only if g is slightly β^* -continuous.

Proof: Let O be a clopen set in (Z, η) . Since, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly β^* -continuous, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Since, f is surjective and pre β^* -open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is β^* -open in Y . Hence, g is slightly β^* -continuous.

Conversely, let g is slightly β^* -continuous. Let O be a clopen set in (Z, η) , then $g^{-1}(O)$ is β^* -open in Y . Since, f is β^* -irresolute, $f^{-1}(g^{-1}(O))$ is β^* -open in X . Hence, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly β^* -continuous.

Theorem 4.21: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a slightly β^* -continuous and (Y, σ) is a locally indiscrete space then f is β^* -continuous.

Proof: Let O be an open subset of Y . Since, (Y, σ) is a locally indiscrete space, O is closed in Y . Since, f is slightly β^* -continuous, $f^{-1}(O)$ is β^* -open in X . Hence, f is β^* -continuous.

Theorem 4.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a slightly β^* -continuous and A is an open subset of X then the restriction $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly β^* -continuous.

Proof: Let V be a clopen subset of Y . Then $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is β^* -open and A is open, $(f|_A)^{-1}(V)$ is β^* -open in the relative topology of A . Hence, $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly β^* -continuous.

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