

FIBONACCI SQUARE SUM SLITHER LABELING OF GRAPHS

R. SREENIVASAN

ASSISTANT PROFESSOR, POST GRAUDATE AND DEPARTMENT OF MATHEMATICS,
AGURCHAND MANMULL JAIN COLLEGE, MEENAMBAKKAM, CHENNAI, INDIA

ABSTRACT: In Mathematics, the idea of Fibonacci numbers given as a sequence of numbers which is obtained by adding the previous two numbers. The Fibonacci numbers begins with 0 and 1. The concept of Fibonacci labeling denotes the assignment of integers to the vertices or edges or both using the Fibonacci numbers. The applicability of Fibonacci numbers to some graphs are studied in this paper.

KEYWORDS: Fibonacci labeling, Square Sum Labeling, Slither labeling, Broom Graph, Double Broom Umbrella Graph and Caterpillar Graph.

INTRODUCTION: The application of Fibonacci numbers in modern day has spread far and wide that ranges from the field of trading, computer algorithms and software design. The Fibonacci numbers also play a pivotal role in the growth patterns in flowers etc. The gamut of applications of Fibonacci numbers have provided the impetus to try the Fibonacci labeling for a different types of graphs.

IMPORTANT DEFINITIONS:

FIBONACCI NUMBERS: The Fibonacci numbers begin with 0 and 1. The subsequent numbers are generated by adding the prior two numbers in the sequence.

SQUARE SUM LABELING: The notion of square sum labeling that is under consideration in this paper is due to Mitesh J. Patel and G.V. Ghodasara [2] who define the same as follows; “A graph G on p vertices and q edges is said to be square sum labelled graph if there is a leeway to have a bijective map $z:V(G) \rightarrow \{0,1,2,\dots,p-1\}$ such that the induced function $z^*:E(G) \rightarrow \mathbb{N}$ given by $z^*(uv) = (z(u))^2 + (z(v))^2$, for each edge $uv \in E(G)$ is an injective map”.

SLITHER LABELING: The idea of labeling is the allocation of numbers to the vertices or edges or both in a graph. Slither labeling is a manipulation of labeling is the assignment of numbers in the slither pattern.

BROOM GRAPH: A broom graph, denoted by $B(n, m)$ is a graph that is obtained from a path on n vertices having m pendant vertices with each vertex attached to one end of the graph. The number of vertices in the graph is $m + n$ and the number of edges is $m + n - 1$.

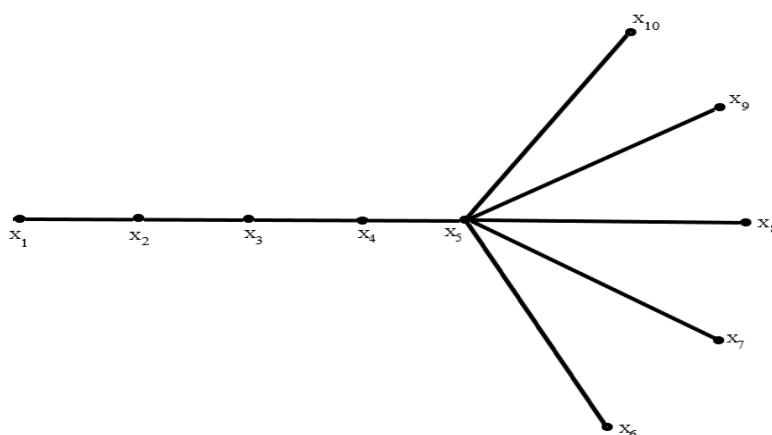


Fig 1. Broom Graph $B(5, 5)$

DOUBLE BROOM GRAPH: The double broom graph $DB(m, n)$ is a special type of broom graph that is obtained from a path on n vertices and has m pendant vertices attached to both the ends of the path graph. The number of vertices in the graph is $2m + n$ and the number of edges is $2m + n - 1$.

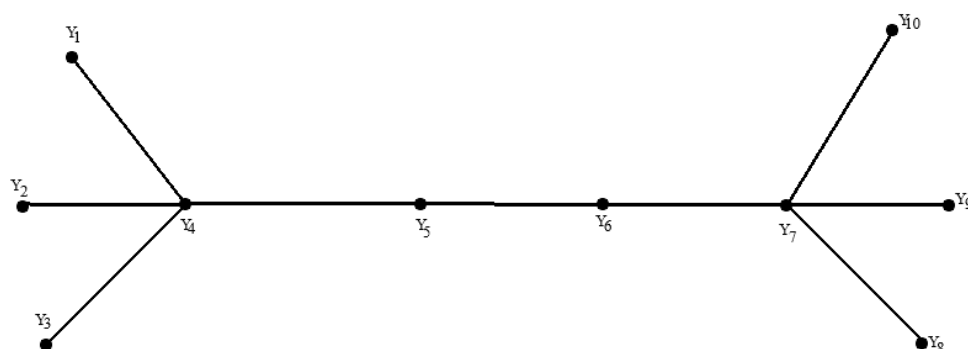


Fig 2. Double Broom Graph $B(3, 4)$

UMBRELLA GRAPH: An umbrella graph $U_{m,n}$ given by Lavanya S and Ganesan V [1] is a special type of path graph that is obtained from a path on n vertices by connecting the central vertex of a fan graph f_m on m vertices with the n^{th} vertex of the path. The number of vertices in the graph is $m + n$ and the number of edges is $2m + n - 2$. Sreenivasan R[4] [5] has proved that the umbrella graph admits vertex antimagic edge slither labeling and square sum slither labeling.

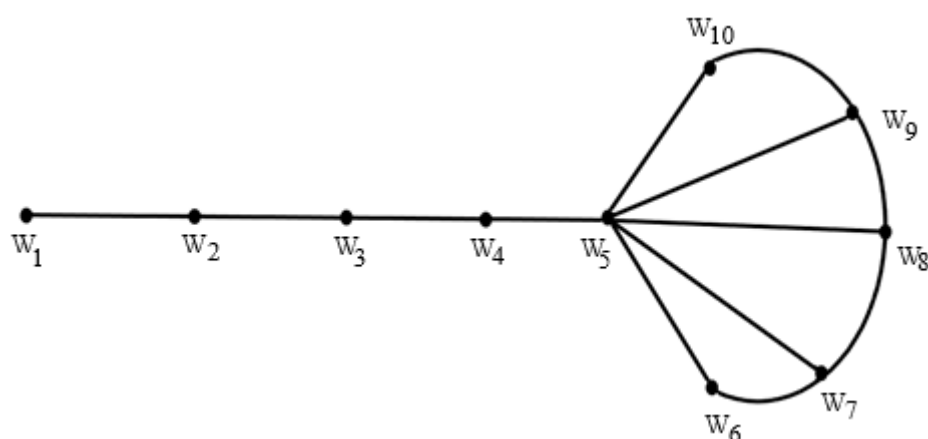


Fig 3. Umbrella Graph $U_{5,5}$

CATERPILLAR GRAPH: A caterpillar graph denoted by P_n^m , that is under consideration in this paper is due to P. Packiavathi, S. Balamurugan, and R. B. Gnanajothi [3], is defined as a graph that is generated from a path P_n and adding x_m pendant vertices to the m^{th} vertex of the path. A caterpillar graph is necessarily a tree with the property that the elimination of the leaves and the incident edges renders the path P_n and is called as the spine of the caterpillar.

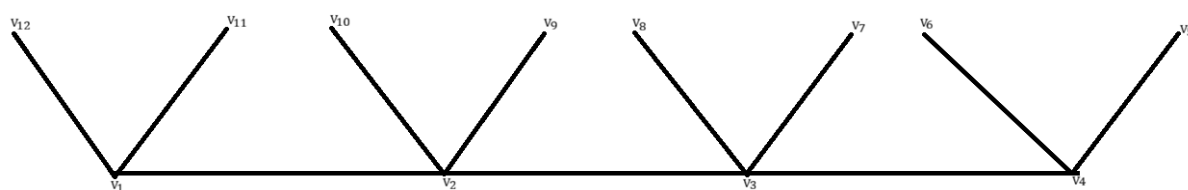


Fig 4. Caterpillar Graph P_4^2

MAIN RESULTS:

Theorem – 1: The broom graph $B(n, m)$ admits Fibonacci Square Sum Slither Labeling.

Proof: Consider the broom graph $B(n, m)$, a graph that is formed on a path of n vertices with m pendant vertices attached to the end vertex of the path. Let $n = 4$ and $m = 4$ be arbitrary values. Slither label the vertices of the graph with Fibonacci numbers in such a way that no vertex labels are repeated. The label of an edge is the sum of squares of labels of the vertices that are incident with it. The Fibonacci square sum slither labeling is given by

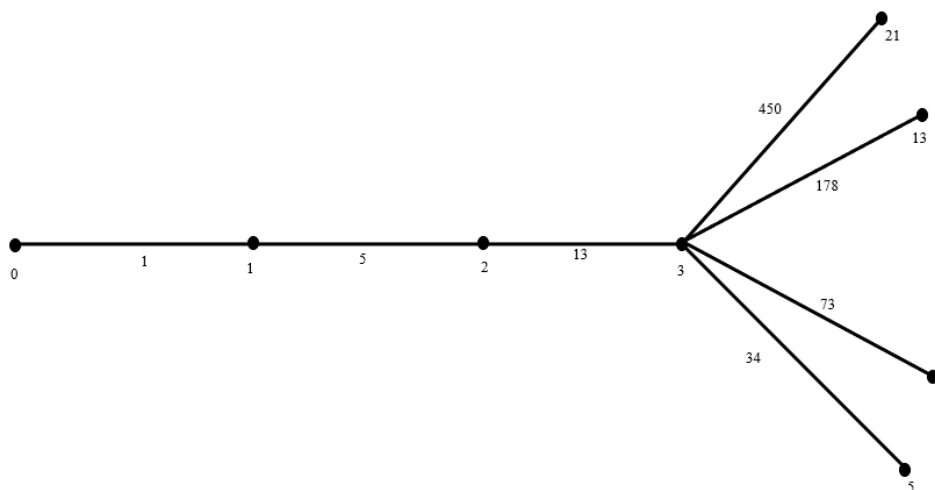


Fig 5. Fibonacci Square Sum Slither Labeling in $B(4, 4)$

In the above broom graph, the labels of the vertices follow slither pattern with all the vertices having unique labeling. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are also unique. Hence the graph admits Fibonacci Square Sum Slither Labeling. Since the values $m = 4$ and $n = 4$ are arbitrary, it follows that the graph admits Fibonacci Square Sum Slither Labeling for all values of m and n .

Theorem – 2: The double broom graph $DB(m, n)$ admits Fibonacci Square Sum Slither Labeling

Proof: Consider the double broom graph $DB(m, n)$, a graph that is formed on a path of n vertices with m pendant vertices attached to the first and the last vertices of the path. Let $n = 3$ and $m = 3$ be arbitrary values. Slither label the vertices of the graph with Fibonacci numbers in such a way that no vertex labels are repeated. The label of an edge is the sum of squares of labels of the vertices that are incident with it. The Fibonacci square sum slither labeling is given by

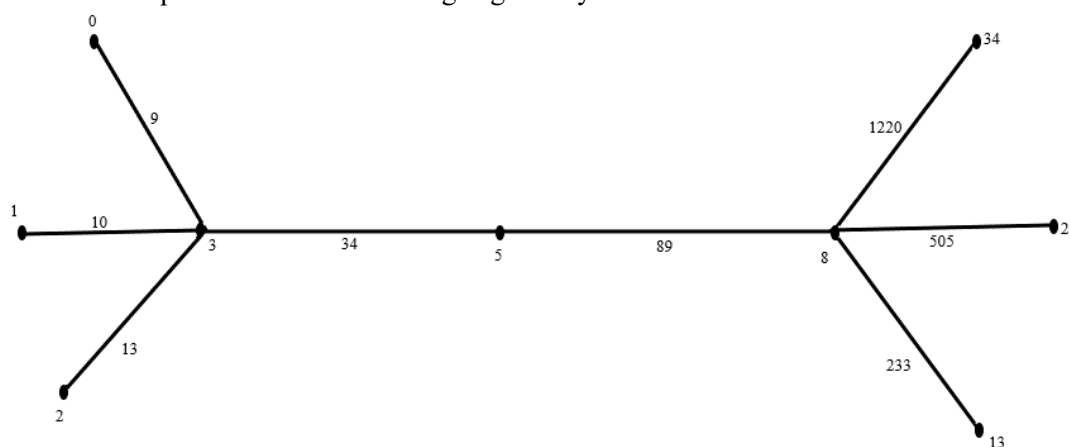


Fig 6. Fibonacci Square Sum Slither Labeling in $DB(3, 3)$

In the above double broom graph, the labels of the vertices follow slither pattern with all the vertices having unique labeling. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are also unique. Hence the graph admits Fibonacci Square Sum Slither Labeling. Since the values $m = 3$ and $n = 3$ are arbitrary, it follows that the graph admits Fibonacci Square Sum Slither Labeling for all values of m and n .

Theorem – 3: The Umbrella Graph admits Fibonacci Square Sum Slither Labeling.

Proof: Consider the umbrella graph $U_{m,n}$, a graph that is from a path on n vertices by connecting the central vertex of a fan graph f_m on m vertices with the n^{th} vertex of the path. Let $n = 4$ and $m = 4$ be arbitrary values. Slither label the vertices of the graph with Fibonacci numbers in such a way that no vertex labels are repeated. The label of an edge is the sum of squares of labels of the vertices that are incident with it. The Fibonacci square sum slither labeling is given by

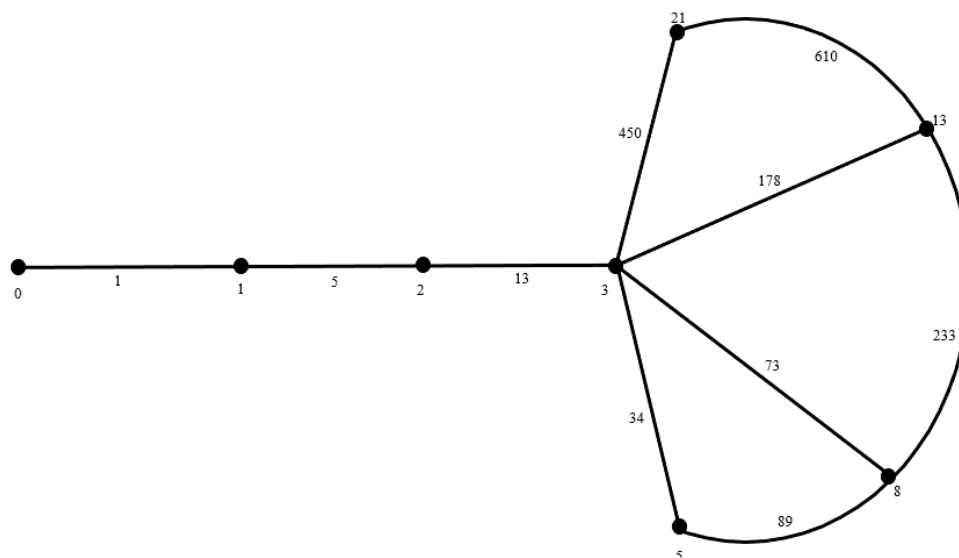


Fig 7. Fibonacci Square Sum Slither Labeling in $U_{4,4}$

In the above umbrella graph, the vertices are labelled follow slither pattern with all labels being unique. The label of an edge is calculated as the sum of squares of labels of the vertices that are incident with it and they are unique as well. Hence the graph admits Fibonacci Square Sum Slither Labeling. Since the values $m = 4$ and $n = 4$ are arbitrary, it follows that the graph admits Fibonacci Square Sum Slither Labeling for all values of m and n .

Theorem – 4: The caterpillar graph admits Fibonacci Square Sum Slither Labeling.

Proof: Consider the caterpillar graph P_n^m , a graph that is generated from a path P_n and adding x_m pendant vertices to the m^{th} vertex of the path. Let $n = 3$ and $m = 3$ be arbitrary values. Slither label the vertices of the graph with Fibonacci numbers in such a way that no vertex labels are repeated. The label of an edge is the sum of squares of labels of the vertices that are incident with it. The Fibonacci square sum slither labeling is given by

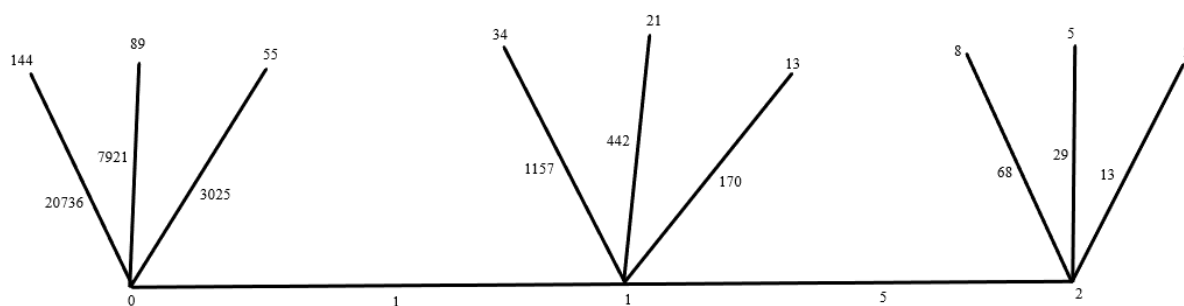


Fig 8. Fibonacci Square Sum Slither Labeling in P_3^3

In the above caterpillar graph, the vertices are labelled follow slither pattern with all labels being unique. The label of an edge is calculated as the sum of squares of labels of the vertices that are incident with it and they are unique as well. Hence the graph admits Fibonacci Square Sum Slither Labeling. Since the values $m = 3$ and $n = 3$ are arbitrary, it follows that the graph admits Fibonacci Square Sum Slither Labeling for all values of m and n .

CONCLUSION: In this paper, three different types of labeling namely, the Fibonacci labeling, Square Sum labeling and Slither labeling are juxtaposed into a single labeling form, Fibonacci Square Sum Slither Labeling and the same has been applied for broom graph, double broom graph, umbrella graph and the caterpillar graph and it has been proved that the above-mentioned graphs admit Fibonacci Square Sum Slither Labeling. In a similar way, different types of labeling can be juxtaposed and can be applied for different graphs.

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