

Development of Fuzzy Intelligent Control for Real Time Pendulum System

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Abstract. Any realistic system model (nonlinear systems) is complex in structure and have large parameter or structural uncertainties, and due to large uncertainties, one cannot sure that such systems can be feedback linearizable. Thus, using T–S fuzzy technique, one can derive a T–S fuzzy system model as an approximation of such systems. The relative degrees and normal forms of such T–S fuzzy systems and the feedback linearization-based control scheme is systematically developed. The feedback linearization control technique for T–S fuzzy systems employs a system normal form based on the system relative degree, and is applicable to systems without restrictive matching conditions that essentially require the local system models to be in some canonical forms. The control law is developed for T-S fuzzy system model and implemented. The simulation results are discussed for inverted pendulum system.

Keywords: Feedback linearization, T-S Fuzzy, intelligent system, tracking, relative degree.

1. Introduction

Nonlinear systems [1], [2] have been successfully modeled and controlled by using fuzzy system techniques. Among various fuzzy techniques, Takagi-Sugeno (T-S) fuzzy system provides desired modeling properties suitable for applying rigorous feedback control designs. In this technique, nonlinear system is approximated in a group of local linear system models in different operation regions and T-S fuzzy model is constructed by combing these linear models with some membership functions [3-5]. A large class of engineering nonlinear systems comprises of mechatronic systems with electrical drives. Such mechatronic systems contain, from a dynamic perspective, one slow subsystem and multiple fast subsystems with significantly smaller time constants [6.7]. A common design strategy for addressing these issues is to linearize the system at a representative operating point. This approximate design approach is simple, but can result in inaccurate or incorrect conclusions owing to its impact on the original characteristics of nonlinear systems [8-10]. Ongoing research in this field, particularly utilizing artificial intelligence methods, is crucial for enhancing the theory and application of linearization and control of nonlinear mechatronic systems. Fuzzy logic-based

linearization can achieve comparable or even superior performance in controlling nonlinear systems while simplifying the linearization process [10, 11].

In contrast to traditional analytical methods, the proposed methodology using fuzzy systems facilitates the development of linearized systems without the need for complex mathematical apparatuses and does not require knowledge of their internal structure and parameters [12-14]. In comparison with linearization methods based on fuzzy logic (data-driven, machine learning), the proposed method reduces computational load because it does not rely on real-time system data or any regression or optimization techniques to ensure the quality of the linearization [21-23]. To control complex nonlinear systems feedback linearization approach can be used that transforms a nonlinear system into a fully or partly linear one. The feedback linearization removes nonlinearities in a nonlinear system ensuring closed loop dynamics in linear form.

However, functions $g(x)$, $f(x)$, $h(x)$ can have un-parameterizable uncertainties. So, one cannot be sure that such a system can be feedback linearizable. Thus, we may not directly and adaptively control the system [20]. Since T-S fuzzy system model can effectively approximate any smooth nonlinear systems to the degree of accuracy one can construct a T-S fuzzy system model as an approximate model of this nonlinear system and such a T-S fuzzy system model can be used as the controlled plant [15-17]. To improve the quality of fuzzy system identification, advanced inference systems such as hybrid fuzzy-neural systems and fuzzy-genetic algorithms have been developed. Hybrid fuzzy-neural systems combine the learning capability of neural networks with the rule-based structure of fuzzy logic [18, 19]. The fuzzy component often requires expert knowledge for rule formation, which may not be readily available in all applications. Hybrid fuzzy-neural systems are widely used in fields such as robotics, process control, and financial modeling.

Traditional fuzzy linearization methods offer flexibility in handling uncertain, nonlinear systems, making them useful when exact models are unavailable, though they may lack the precision of model-based approaches [22, 23]. The implementation difficulty depends on the complexity of the fuzzy rules. In these methods, a continuous nonlinear system is linearized within the fuzzification bands of individual inputs using a linear function is proposed and traditional fuzzy linearization methods offer an alternative to feedback and input-output linearization techniques, which require detailed system models and transformations. These fuzzy methods eliminate the need for precise models. Feedback linearization is highly effective when the system model is accurately known, as it provides exact linearization. However, it becomes less effective in the presence of model inaccuracies or when dealing with highly nonlinear dynamics. The method can be complex to implement and computationally intensive, often requiring the inversion of system dynamics [24, 25].

To deal with the uncertainties of the parameter matrices adaptive control designs are desirable. Adaptive tracking control of nonlinear systems using fuzzy approximation technique has been extensively studied, especially for certain canonical form nonlinear systems. For adaptive state tracking control of T-S fuzzy systems, similar matching conditions are also needed. Restrictive matching conditions lead to major limitations of state tracking adaptive control for T-S fuzzy systems. This limitation can be avoided with the feedback linearization adaptive control of T-S fuzzy system.

2. System description and modelling of CIPS

2.1 Pendulum Set Up

The experimental setup is comprised with a motorized cart mounted on a 1-meter linear track. Pendulums designed for frictionless rotation are attached to the cart's pivot shaft. Horizontal displacement of the cart induces a swinging motion in the pendulums. The mechanical force exerted on the cart is directly proportional to the control voltage supplied to the motor; thus, this voltage serves as the primary control signal. To monitor the system state, optical encoders provide real-time feedback on two critical variables: the angular position of the pendulum and the linear displacement of the cart along the rail. The objective of the control algorithm is to modulate the motor voltage to achieve specific dynamic behaviours, like maintaining an upright equilibrium (stabilization), executing a swing-up manoeuvre, or implementing crane-style load control.

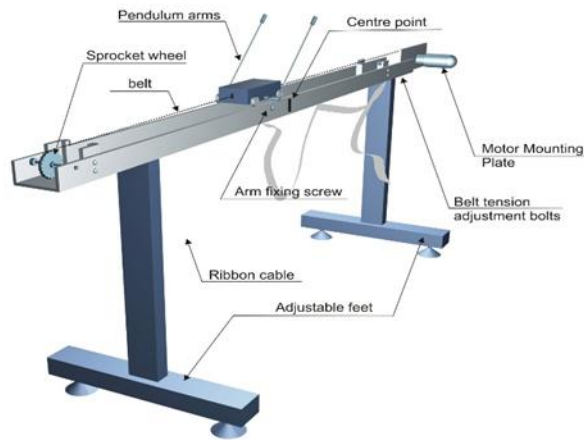


Fig. 1. Digital Pendulum mechanical unit

2.2 Equation of Motion

Based on the system dynamics, the apparatus is characterized as two degrees of freedom (DOF) system with the linear displacement of the cart and the angular oscillation of the pendulum. This motion is a combination of translational movement (defined by the cart's position) and rotational movement (defined by the pendulum's angle).

In this parametric model, the system state is defined by two primary variables, x as the horizontal distance from the Y axis and θ is the angular displacement relative to the vertical axis.

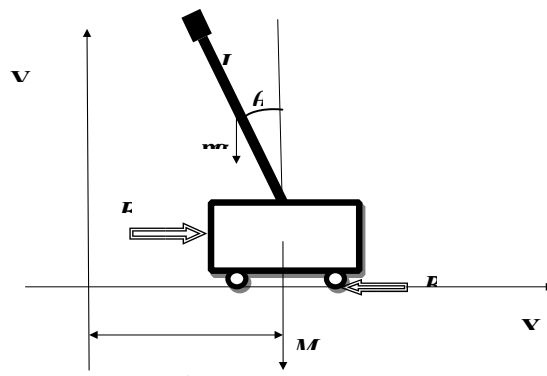


Fig. 2. Parametric representation of the Inverted Pendulum System

The physical properties and variable of inverted pendulum of Fig. 2 are mentioned in table 1. The vertical axis serves as the primary reference for both the linear position of the cart and the angular displacement of the pendulum. For the purpose of mathematical modelling, the pendulum and its supporting rod are treated as a single rigid body. This simplification is justified because both components share identical kinematic characteristics and rotate about the same pivot point. The list of parameters used in the derivation of Inverted Pendulum dynamics are mentioned in table 1.

Table 1. List of parameters used in the derivation of Inverted Pendulum dynamics

Parameter	Value	Unit	Symbol
Mass of the motorized cart	2.4	Kg	M
Distance from the pivot to the centre of gravity	0.4	m	l
Mass of the pendulum	0.23	Kg	m
Moment of inertia of mass of pendulum assembly	0.099	kg/s ²	I
Acceleration due to gravity	9.81	m/ s ²	G
Pendulum damping coefficient	0.005	N-ms/rad	D
Viscous friction coefficient of the cart	0.05	Ns/m	b

The following nonlinear equations of motion is obtained by adding the forces acting on the pendulum and cart system and the moments:

$$F = (m + M)\ddot{X} + b\dot{X} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \quad (1)$$

$$0 = (I + ml^2)\ddot{\theta} + ml\ddot{X}\cos\theta + d\dot{\theta} - mgl\sin\theta \quad (2)$$

2.3 Linear Mathematical Model

The inverted pendulum system is characterized by two distinct equilibrium states: the stable equilibrium (where the pendulum hangs downward) and the unstable equilibrium (where the pendulum is balanced in the upright position). Because the primary control objective is to maintain this upright orientation, the mathematical analysis focuses on the latter. To derive a linearized model at this unstable point, we assume the angular displacement θ remains within a very small range relative to the vertical. Under this small-angle approximation, the system dynamics are expressed as:

At up position i.e. $\theta = 0$, Then $\sin\theta = \theta$ and $\cos\theta = 1$

$$\frac{\theta(s)}{F(s)} = \frac{-0.092s}{0.34869s^3 + 0.14895s^2 + 2.3733s - 0.0451} \quad (3)$$

$$\frac{X(s)}{F(s)} = \frac{0.1358s^2 + 0.005s - 0.9025}{0.34869s^4 + 0.014895s^3 - 2.3733s^2 - 0.0451s} \quad (4)$$

At downward position i.e. $\theta = \pi$ Then $\sin\theta = -\theta$ $\cos\theta = -1$ $\dot{\theta}^2 = 0$

$$\frac{\theta(s)}{F(s)} = \frac{0.092s}{0.34869s^3 + 0.14895s^3 + 2.3733s + 0.0457} \quad (5)$$

$$\frac{X(s)}{F(s)} = \frac{0.1358s^2 + 0.005s + 0.9025}{0.3487s^4 + 0.01499s^2 + 2.37338s^2 - 0.0451s} \quad (6)$$

Thus, now linearized model becomes

At $\theta = 0$

$$\begin{bmatrix} \ddot{X} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0195 & 0.2381 & 0.0013 \\ 0 & 0 & 0 & 1 \\ 0 & 0.0132 & 6.8073 & -0.0377 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3895 \\ 0 \\ -0.2638 \end{bmatrix} U$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (7)$$

At $\theta = \Pi$

$$\begin{bmatrix} \ddot{X} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.019472 & -0.2381 & -0.0013 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0132 & -6.8073 & -0.0377 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3895 \\ 0 \\ 0.2638 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (8)$$

3. T-S Fuzzy System

Consider a single input single output continuous time nonlinear system

$$\begin{aligned} \dot{x} &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \quad (9)$$

where $x(t) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}$ is the system input and $y(t) \in \mathbb{R}$ is the system output, and $f(x(t)) \in \mathbb{R}^n$, $g(x(t)) \in \mathbb{R}^n$ and $h(x(t)) \in \mathbb{R}$ are all unknown smooth functions of $x(t)$. A T–S fuzzy system model can effectively approximate a nonlinear system.

Using a standard T–S fuzzy modeling technique, the local fuzzy logic rules

R_i , $i=1, 2, \dots, N$, As

R_i : If $Z_1(t) \in F_{i1}$ and $Z_L(t) \in F_{iL}$

Then

$$\dot{x} = A_i x(t) + B_i u(t), y(t) = C_i x(t) \quad (10)$$

Where $Z_j(t)$, $j=1, 2, \dots, L$ are the premise variables which are some or all of the state variables F_j is the premise variable interval, and there is a membership function F_{ij} to measure the degree of $Z_j(t)$. Based on these local fuzzy models, a commonly used global T–S fuzzy system model is

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^N \mu_i A_i x(t) + \sum_{i=1}^N \mu_i B_i u(t) \\ y(t) &= \sum_{i=1}^N \mu_i C_i x(t) \end{aligned} \quad (11)$$

where $\mu_i = \frac{\lambda_i}{\lambda}$ with $\lambda = \sum_{i=1}^N \mu_i \lambda_i$, $\lambda_i = \prod_{j=1}^L F_{ij}$, such that $\mu_i \in [0, 1]$ and $\sum_{i=1}^N \mu_i = 1$

Eq. (11) is a natural choice to derive a global T–S fuzzy model, and it proves to be effective

and successful in most applications. The function μ_i measures the degree of the i^{th} subsystem in the global system (9). Since the membership functions F_{ij} are functions of the premise variables Z_j such as triangular, trapezoid, or Gaussian functions.

For many applications, the system output $y(t)$ is a linear combination of $x(t)$, that is $y(t)=C x(t)$ for some constant matrix C , the T-S fuzzy system model is

$$\begin{aligned}\dot{x}(t) &= \sum_{i=0}^N \mu_i A_i x(t) + \sum_{i=0}^N \mu_i B_i u(t) \\ y(t) &= C x(t)\end{aligned}\quad (12)$$

3.1 Fuzzy Rules:

Fuzzy rules are designed based on two equilibrium positions as discussed. These rules are as follows

- If $x_1(t)$ is about to 0 then $\dot{x}(t) = A_1 x(t) + B_1 u(t)$
 - If $x_1(t)$ is about to $\pm\pi$ then $\dot{x}(t) = A_2 x(t) + B_2 u(t)$
- Where $x_1(t)$ is the pendulum angle (theta)

3.2 Fuzzy Membership Function:

The shown triangular membership functions in Fig. 3, is for pendulum angle (Theta). Angular motion varies from -180 degree to +180 degree (-3.14 radian to +3.14 radians). Three triangular membership functions are used

1. $Mf1$: -3.14 to -3.14 to 0,
2. $Mf2$: -3.14 to 0 to +3.14,
3. $Mf3$: 0 to +3.14 to +3.14.

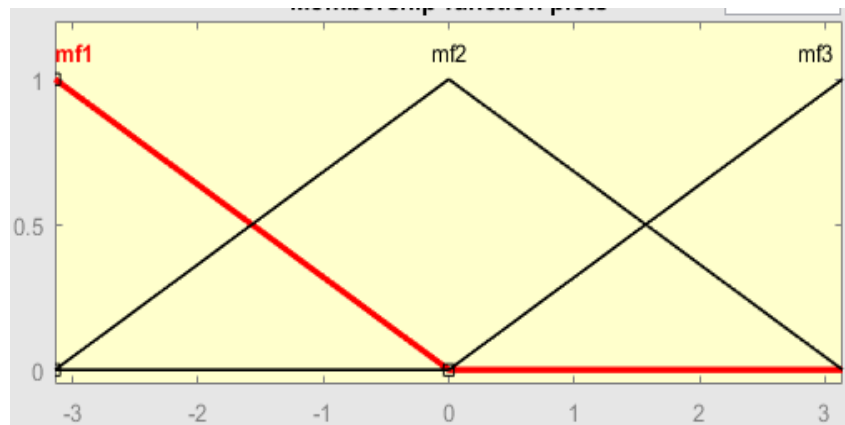


Fig. 3. Fuzzy membership function for pendulum angle

T-S fuzzy system is designed with two rules as shown in Fig.4. System variable angle (theta) is given to three triangular functions. w_1 weight is derived from triangular function 1(Rule 1). w_2 weight is derived from triangular function 2 (Rule 2). These weights play very important role in T-S fuzzy model of any nonlinear real time system. According to Eq. (13), these weights are multiplied with state matrix A and B and T-S Fuzzy model is derived. The results of this model are compared with the real time system.

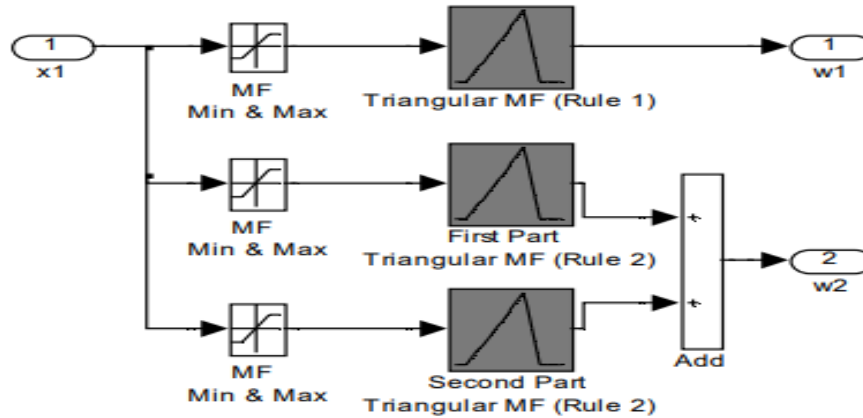


Fig. 4. Fuzzy Operational Model

4. FEEDBACK LINERIZATION OF T-S FUZZY SYSTEM

4.1 Relative degree of T-S Fuzzy System:

Consider the T–S fuzzy system,

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^N \mu_i A_i x(t) + \sum_{i=1}^N \mu_i B_i u(t) \\ y(t) &= Cx(t) \end{aligned} \quad (13)$$

The T–S fuzzy system has uniform relative degree 1 if and only if

$$\sum_{i=1}^N \mu_i C B_i \neq 0 \quad \forall x \in R^n$$

The T–S fuzzy system has uniform relative degree 2 if and only if

$$\sum_{i=1}^N \mu_i C B_i = 0 \quad \forall x \in R^n$$

$$\sum_{i=1}^N \sum_{j=1}^N \mu_i \mu_j C A_i B_j + \sum_{i=1}^N \sum_{j=1}^N C A_i \frac{\partial \mu_i}{\partial x} \mu_j B_j \neq 0 \quad \forall x \in R^n \quad (14)$$

For control systems with relative degree 1

$$\dot{y}(t) = R_1(x) + G_1(x)u(t) \quad (15)$$

$$\text{Where } R_1 = \sum_{i=1}^N \mu_i C A_i x \quad \text{and} \quad G_1 = \sum_{i=1}^N \mu_i C B_i \neq 0$$

When T-S fuzzy system has uniform relative degree ρ , the system can be transformed into two subsystems via diffeomorphism: tracking dynamics subsystem and the zero dynamics subsystem which has the form

$$\begin{aligned} y &= \varepsilon_1 \\ \varepsilon_l &= \varepsilon_{l+1}, l = 1, 2, \dots, \rho - 1. \\ \dot{\varepsilon}_\rho &= R_\rho(x) + G_\rho(x)u \end{aligned} \quad (16)$$

The zero dynamics subsystem is

$$\eta = q(\varepsilon, \eta)$$

For relative degree=1,

$$R_1 = \sum_{i=1}^N \mu_i C A_i x \text{ and } G_1 = \sum_{i=1}^N \mu_i C B_i \neq 0$$

$$\dot{y}(t) = R_1(x) + G_1(x)u(t)$$

The control law will be

$$u(t) = \frac{1}{G_1(t)} (-R_1(t)x(t) - \alpha y(t) + \alpha y_m(t) + \dot{y}_m(t)) \quad (17)$$

Where $\alpha > 0$ is a design parameter and y_m is a given reference output signal which is bounded and has a bounded first order derivative ($\dot{y}_m$)

5. SIMULATION AND RESULT

5.1 CIPS system model

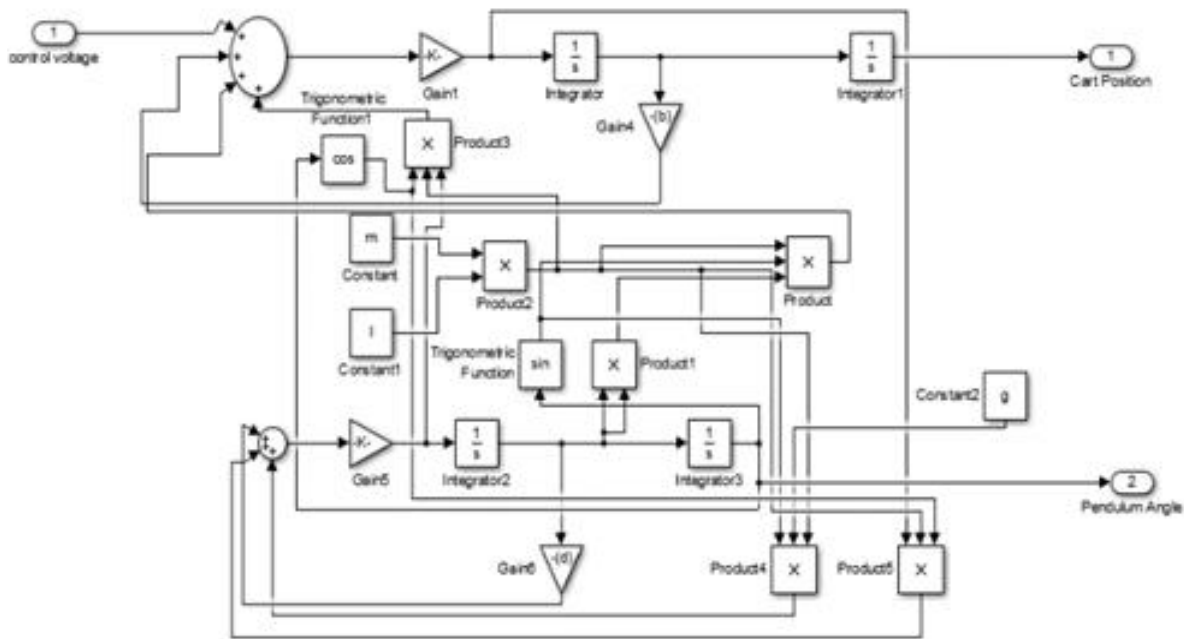


Fig. 5. CPIS System Simulink Model

The system shows Simulink model of CIPS system implemented with MATLAB. In this model is implemented and behavior of CIPS system is observed with system parameters cart position and angle. Fig. 6 shows the output of CIPS system Simulink model 1. cart position 2. Pendulum angle without controller. The output of CIPS system shows that the cast position is increasing and angle of pendulum is continuously oscillating. Output of CIPS without controller

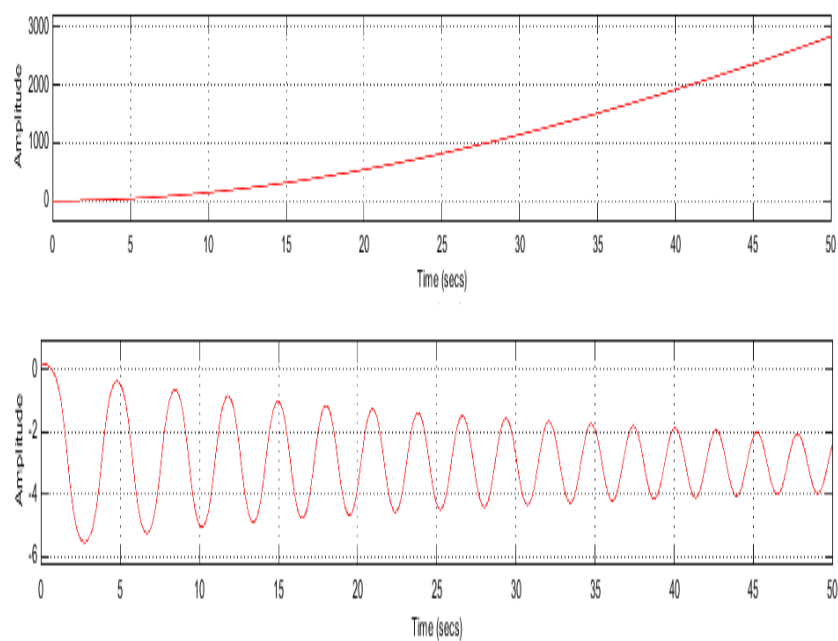


Fig. 6. Output of CIPS Simulink Model (without controller)

5.2 Linearized CIPS Model

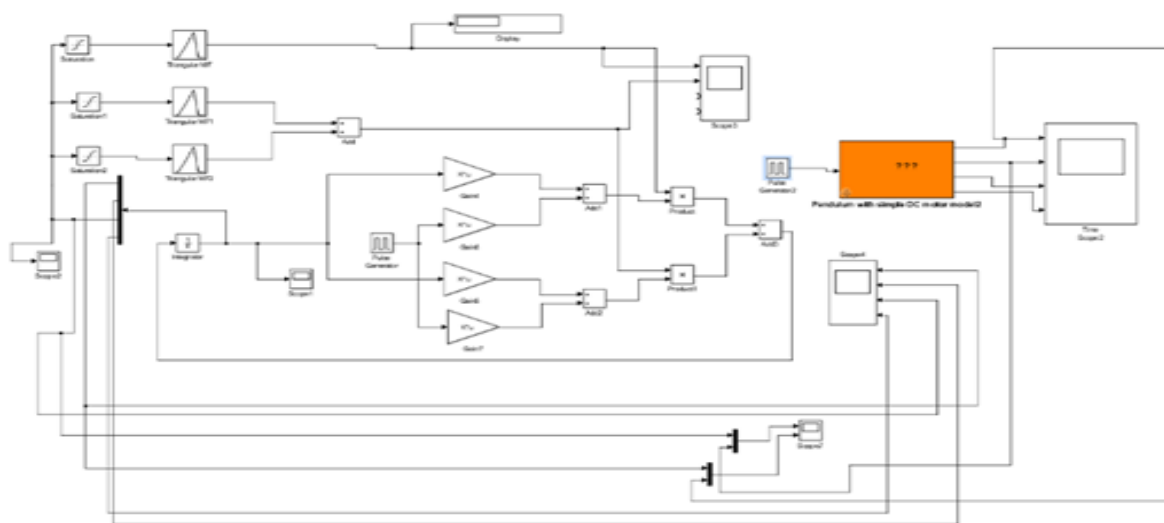


Fig. 7. Simulink Model of Linearized T-S Fuzzy Model of CIPS.

The linearized model of CIPS system and membership function are implemented with the help of MATLAB Simulink. Triangular membership function is used as shown in Fig. 3. Two rule membership functions are used as per Fig. 4. fuzzy operational model block. Simulation result of this model gives result as shown in Fig.8. It shows that the designed linearized model of CIPS matches result of original results of CIPS (Cart position and angle).

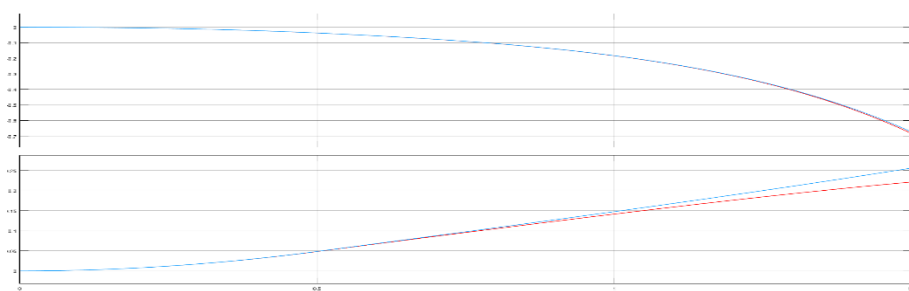


Fig. 8. Simulation of Linearized CIPS Model (Without controller).

5.3 CIPS with T-S Fuzzy Feedback Linearization Controller

For relative degree one, the control design has the following details. For Nominal control scheme: To illustrate the basic feedback linearization control, control design is presented when system parameters (A_i , B_i , C) are known as the nominal control scheme. When System 4.3 has uniform relative degree 1, we have

$$\dot{y}(t) = R_1(x) + G_1(x)u(t)$$

$$R_1 = \sum_{i=1}^N \mu_i CA_i x \quad \text{and} \quad G_1 = \sum_{i=1}^N \mu_i CB_i \neq 0 \quad (18)$$

If the Eq. (18) has uniform relative degree 1, CB_i , $i=1, \dots, N$ are all strictly positive or negative. To achieve the desired output, the nominal control law will be

$$u(t) = \frac{1}{G_1(t)} (-R_1(t)x(t) - \alpha y(t) + \alpha y_m(t) + \dot{y}_m(t)) \quad (19)$$

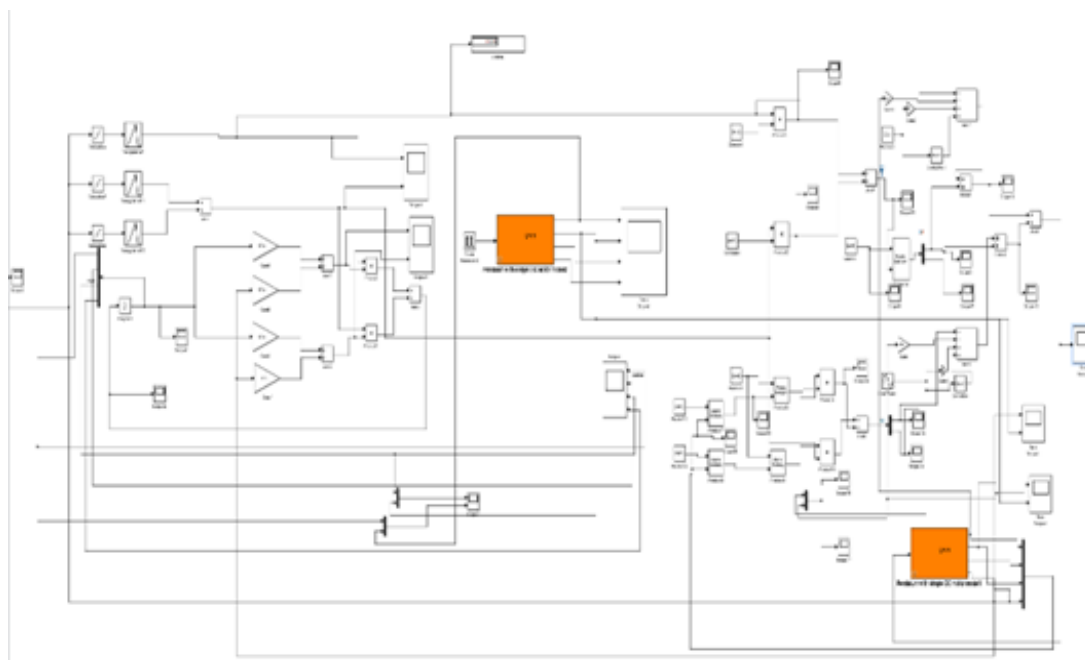


Fig. 9. Feedback Linearization Controller.

Fig. 9. shows the MATLAB Simulation of Feedback Linearization Controller for T-S Fuzzy Model of CIPS System. Eq. (18) and Eq. (19) are implemented with the help of MATLAB. The proposed controller design is verified for two cases.

5.4 CIPS with T-S Fuzzy Feedback Linearization Controller response

When any external disturbance is given to the pendulum system, controller will maintain the cart position at one position and angle settled to zero within 6 sec. Control signal generated by the controller is shown in Fig. 12.

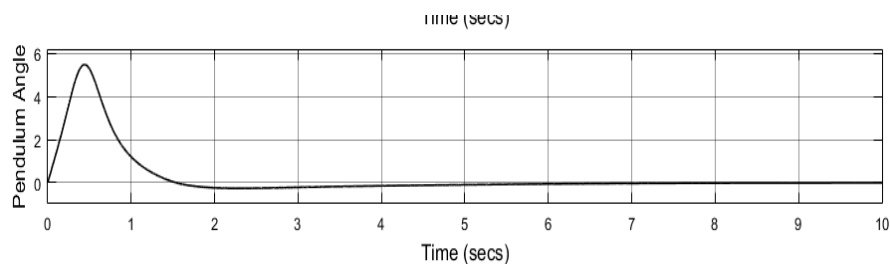


Fig. 10. Pendulum angle with designed controller.

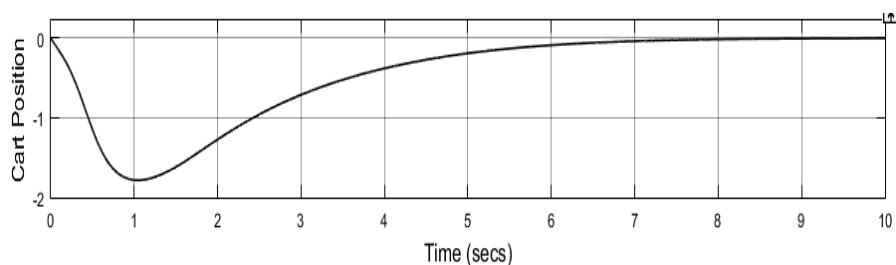


Fig. 11. Cart position with designed controller.

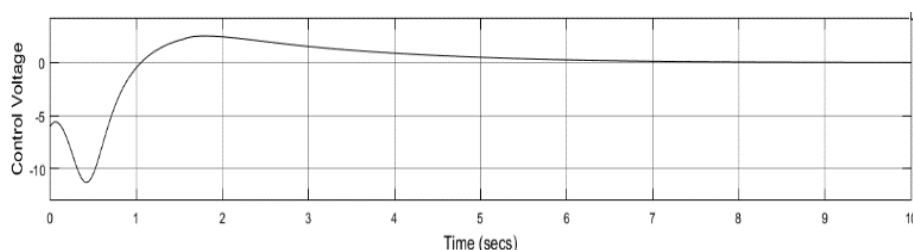


Fig. 12. Control input with designed controller.

5.5 CIPS with T-S Fuzzy Feedback Linearization Controller response (Sinusoidal input)

When any sinusoidal disturbance is given to the pendulum system, controller will maintain the cart position at one position with oscillations of small amplitude and angle settled to zero within 6 secs with oscillations of small amplitude. Control signal generated by the controller is shown in Fig. 15.

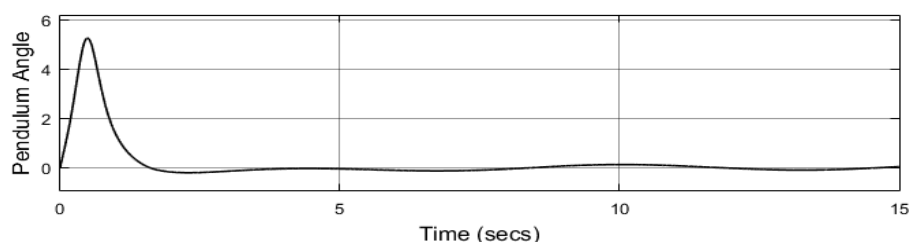


Fig. 13. Pendulum angle with designed controller.

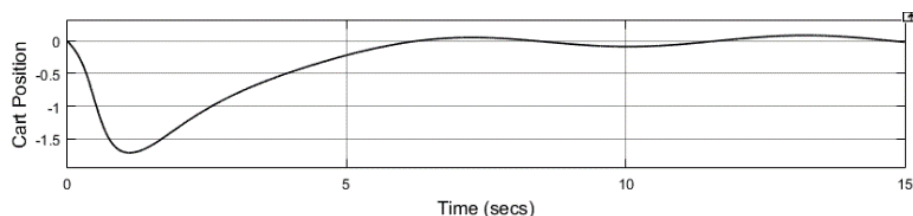


Fig. 14. Cart position with designed controller.

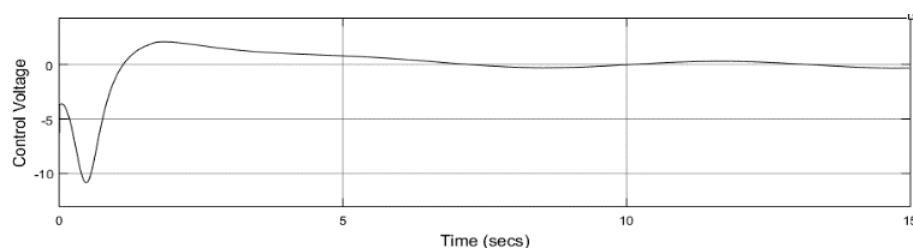


Fig. 15. Control input with designed controller.

Conclusion

In this research work, two controllers are designed PID controller and Feedback Linearization Controller for T-S Fuzzy System. PID Controller settle the pendulum angle to zero within 6 sec. while Feedback Linearization Controller for T-S Fuzzy System settle the pendulum angle to zero within 5 to 6 sec. In conclusion, the feedback linearization of T-S fuzzy systems offers a compelling avenue for enhancing control performance. By leveraging the synergies between feedback control and fuzzy logic, this approach paves the way for adaptive and robust control solutions in diverse applications. The implications and applications of feedback linearization of T-S fuzzy systems extend across diverse domains, including automation, robotics, aerospace, and beyond. This approach holds promise for addressing complex control challenges in various industries.

Acknowledgement

Authors are thankful to the Department of Electrical Engineering, Walchand College of Engineering, Sangli for extending their facilities for conduction of this research work.

Conflict of Interest

This research work has no conflict of interest.

Author Contribution

Author 1: Concept, Experimentation, result analysis, Drafting of manuscript

Author 2: Experimentation, manuscript editing

Author 3: Result validation, manuscript editing

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